

1⁻⁻ and 0⁺⁺ heavy four-quark and molecule states in QCD

R.M. Albuquerque^{a,b,1,*}, F. Fanomezana^c, S. Narison^b, A. Rabemananjara^c

^aInstituto de Física, Universidade de São Paulo, C.P. 66318, 05389-970 São Paulo, SP, Brazil

^bLaboratoire Particules et Univers de Montpellier, CNRS-IN2P3, Case 070, Place Eugène Bataillon, 34095 - Montpellier, France.

^cInstitute of High-Energy Physics of Madagascar (iHEP-MAD), University of Antananarivo, Madagascar

Abstract

We estimate the masses of the 1⁻⁻ heavy four-quark and molecule states by combining exponential Laplace (LSR) and finite energy (FESR) sum rules known perturbatively to lowest order (LO) in α_s but including non-perturbative terms up to the complete dimension-six condensate contributions. We use double ratio of sum rules (DRSR) for determining the $SU(3)$ breakings terms. The $SU(3)$ mass-splittings of about (50 – 110) MeV and the ones of about (250 – 300) MeV between the lowest ground states and their 1st radial excitations are (almost) heavy-flavour independent. The mass predictions summarized in Table 2 are compared with the ones in the literature (when available) and with the three $Y_c(4260, 4360, 4660)$ and $Y_b(10890)$ 1⁻⁻ experimental candidates. We conclude that the lowest observed state cannot be a *pure* 1⁻⁻ four-quark nor a *pure* molecule but may result from their mixings. We extend the above analyzes to the 0⁺⁺ four-quark and molecule states which are about (0.5-1) GeV heavier than the corresponding 1⁻⁻ states, while the splittings between the 0⁺⁺ lowest ground state and the 1st radial excitation is about (300-500) MeV. We complete the analysis by estimating the decay constants of the 1⁻⁻ and 0⁺⁺ four-quark states. Our predictions can be tested using some alternative non-perturbative approaches or/and at LHC_b or some other hadron factories.

Keywords: QCD spectral sum rules, four-quark and molecule states, heavy quarkonia.

1. Introduction and a short review on the 1⁺⁺ channel

A large amount of exotic hadrons which differ from the “standard” $\bar{c}c$ charmonium and $\bar{b}b$ bottomium radial excitation states have been recently discovered in B -factories through $J/\psi\pi^+\pi^-$ and $\Upsilon\pi^+\pi^-$ processes and have stimulated different theoretical interpretations. Most of them have been assigned as four-quarks and/or molecule states [1]. In previous papers [2, 3], some of us have studied, using exponential QCD spectral sum rules (QSSR) [4–6] and the double ratio of sum rules (DRSR) [7–10], the nature of the $X(3872)$ 1⁺⁺ states found by Belle [11] and confirmed by Babar [12], CDF [13] and D0 [14]. If it is a $(cq)(\bar{c}q)$ four-quark or $D - D^*$ molecule state, one finds [2]:

$$X_c = (3925 \pm 127) \text{ MeV}, \text{ with } \sqrt{t_c} = (4.15 \pm 0.03) \text{ GeV} \quad (1)$$

corresponding to a t_c -value common solution of the exponential Laplace (LSR) and Finite Energy (FESR) sum rules. While in the b -meson channel, using $m_b = 4.26$ GeV, one finds [2]:

$$X_b = (10144 \pm 104) \text{ MeV}, \text{ with } \sqrt{t_c} = (10.4 \pm 0.02) \text{ GeV}. \quad (2)$$

By assuming that the mass of the radial excitation $X'_Q \approx \sqrt{t_c}$, one can also deduce the mass-splitting:

$$X'_c - X_c \approx 225 \text{ MeV} \approx X'_b - X_b \approx 256 \text{ MeV}, \quad (3)$$

which is much lower than the ones of ordinary $\bar{c}c$ and $\bar{b}b$ states:

$$\psi(2S) - \psi(1S) \approx 590 \approx \Upsilon(2S) - \Upsilon(1S) \approx 560 \text{ MeV}, \quad (4)$$

suggesting a different dynamics for these exotic states.

2. QCD Analysis of the 1⁻⁻ and 0⁺⁺ channels

In the following, we extend the previous analysis to the case of the 1⁻⁻ and 0⁺⁺ channels and improve some existing analysis from QCD (spectral) sum rules in the 1⁻⁻ channel [15, 16]. The results will be compared with the experimental 1⁻⁻ candidate states: $Y(4260)$, $Y(4360)$, $Y(4660)$, $Y_b(10890)$. These states have been seen by Babar [17] and Belle [18, 19] and which decay into $J/\psi\pi^+\pi^-$ and $\Upsilon\pi^+\pi^-$ around the $\Upsilon(5S)$ mass. These states cannot be identified with standard $\bar{c}c$ charmonium and $\bar{b}b$ bottomium radial excitations and have been assigned to be four-quark or molecule states or some threshold effects.

• Interpolating currents

We assume that the Y states are described either by the lowest dimension (without derivative terms) four-quark and molecule $\bar{D}_s D_s^*$ vector currents J^μ given by:

$$J_{4q}^\mu = \frac{\epsilon_{abc}\epsilon_{dec}}{\sqrt{2}} \left\{ \left[\left(s_a^T C \gamma_5 Q_b \right) \left(\bar{s}_d \gamma^\mu \gamma_5 C \bar{Q}_e^T \right) + \left(s_a^T C \gamma_5 \gamma^\mu Q_b \right) \left(\bar{s}_d \gamma_5 C \bar{Q}_e^T \right) \right] + b \left[\left(s_a^T C Q_b \right) \left(\bar{s}_d \gamma^\mu C \bar{Q}_e^T \right) + \left(s_a^T C \gamma^\mu Q_b \right) \left(\bar{s}_d C \bar{Q}_e^T \right) \right] \right\} \quad (5)$$

$$J_{mol}^\mu = \frac{1}{\sqrt{2}} \left(\frac{g'}{\Lambda'^2} \right)_{\text{eff}}^2 \left[\left(\bar{s} \gamma^\mu Q \right) \left(\bar{Q} s \right) + \left(\bar{Q} \gamma^\mu s \right) \left(\bar{s} Q \right) \right] \quad (6)$$

*Speaker, PhD Student fellow, FAPESP CNPq-Brasil.

Email addresses: rma@if.usp.br (R.M. Albuquerque), fanfenos@yahoo.fr (F. Fanomezana), snarison@yahoo.fr (S. Narison), achriss_01@yahoo.fr (A. Rabemananjara)

• QCD input parameters

The QCD parameters are given in Table 1 and we shall work with the running light quark parameters [20, 21].

Table 1: QCD input parameters.

Parameters	Values	Ref.
$\Lambda(n_f = 4)$	$(324 \pm 15) \text{ MeV}$	[22–24]
$\Lambda(n_f = 5)$	$(194 \pm 10) \text{ MeV}$	[22–24]
\hat{m}_s	$(0.114 \pm 0.021) \text{ GeV}$	[5, 24]
m_c	$(1.26 \sim 1.47) \text{ GeV}$	[5, 24–27]
m_b	$(4.17 \sim 4.70) \text{ GeV}$	[5, 24–26]
$\hat{\mu}_q$	$(263 \pm 7) \text{ MeV}$	[5]
$\kappa \equiv \langle \bar{s}s \rangle / \langle \bar{u}u \rangle$	(0.74 ± 0.06)	[9]
M_0^2	$(0.8 \pm 0.2) \text{ GeV}^2$	[28–30]
$\langle \alpha_s G^2 \rangle$	$(7 \pm 2) \times 10^{-2} \text{ GeV}^4$	[22, 26, 31–37]
$\langle g^3 G^3 \rangle$	$(8.3 \pm 1.0) \text{ GeV}^2 \times \langle \alpha_s G^2 \rangle$	[26]
$\rho \equiv \langle \bar{q}q\bar{q}q \rangle / \langle \bar{q}q \rangle^2$	(2 ± 1)	[22, 28, 31]

3. 1^{--} four-quark state mass Y_{Qq} from QSSR

In the following, we shall estimate the mass of the 1^{--} four-quark state $(\bar{Q}q)(Qq)$, hereafter denoted by Y_{Qd} . In so doing, we shall use the ratios of the Laplace (exponential) sum rule and of FESR:

$$\mathcal{R}_{Qd}^{LSR}(\tau) \simeq M_{Y_{Qd}}^2 \simeq \mathcal{R}_{Qd}^{FESR}. \quad (7)$$

• The Y_{cd} mass from LSR and FESR for the case $b=0$

Using the QCD inputs in Table 1, we show the τ -behaviour of $M_{Y_{cd}}$ from \mathcal{R}_{cd}^{LSR} in Fig. 1a. One can notice from Fig. 1a that the τ -stability is obtained from $\sqrt{t_c} \geq 5.1 \text{ GeV}$, while the t_c -stability is reached for $\sqrt{t_c} = 7 \text{ GeV}$. The most conservative prediction from the LSR is obtained in this range of t_c -values for $m_c = 1.26 \text{ GeV}$ and gives in units of GeV:

$$\begin{aligned} 4.79 \leq M_{Y_{cd}} \leq 5.73 & \text{ for } 5.02 \leq \sqrt{t_c} \leq 7 \text{ and } m_c = 1.26, \\ 5.29 \leq M_{Y_{cd}} \leq 6.11 & \text{ for } 5.5 \leq \sqrt{t_c} \leq 7 \text{ and } m_c = 1.47. \end{aligned}$$

We compare in Fig. 1b), the t_c -behaviour of the LSR results obtained at the τ -stability points with the ones from \mathcal{R}_{cd}^{FESR} for $m_c=1.23 \text{ GeV}$ (running) and 1.47 GeV (on-shell). One can deduce the common solution in units of GeV:

$$\begin{aligned} M_{Y_{cd}} &= 4.814 \text{ for } \sqrt{t_c} = 5.04(5) \text{ and } m_c = 1.26, \\ &= 5.409 \text{ for } \sqrt{t_c} = 5.6 \text{ and } m_c = 1.47. \end{aligned} \quad (8)$$

We observe that the on-shell c -quark mass value tends to overestimate $M_{J/\psi}$ [3, 26]. The same feature happens for the evaluation of the $X(1^{++})$ four-quark state mass [2]. Then, we are tempted to take as a final result in this paper the prediction obtained by using the running mass $\bar{m}_c(m_c) = 1262(17) \text{ MeV}$. Considering the uncertainties from the Table (1), we deduce

$$M_{Y_{cd}} = 4814(57) \text{ MeV}. \quad (9)$$

Using the fact that the 1st FESR moment gives a correlation between the mass of the lowest ground state and the onset of continuum threshold t_c , we shall approximately identify its value with the one of the radial excitation. Assuming that, one can deduce the mass-splitting: $M'_{Y_{cd}} - M_{Y_{cd}} \approx 226 \text{ MeV}$, which is similar to the one obtained for the $X(1^{++})$ four-quark state [2].

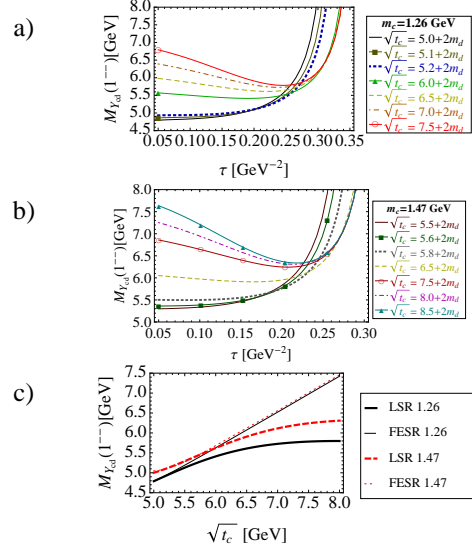


Figure 1: a) τ -behaviour of $M_{Y_{cd}}(1^{--})$ from \mathcal{R}_{cd}^{LSR} for the current mixing parameter $b = 0$, for different values of t_c and for $m_c = 1.26 \text{ GeV}$; b) The same as a) but for $m_c = 1.47 \text{ GeV}$; c) t_c -behaviour of the LSR results obtained at the τ -stability points and comparison with the ones from \mathcal{R}_{cd}^{FESR} for $m_c = 1.26$ and 1.47 GeV .

• The Y_{bd} mass from LSR and FESR for the case $b = 0$

We extend the previous analysis to the b -quark sector, in order to estimate the mass of Y_{bd} four-quark state. Considering, like in the case of charm, as a final estimate the one from the running b -quark mass $\bar{m}_b(m_b) = 4177(11) \text{ MeV}$ [26], we deduce:

$$M_{Y_{bd}} = 11256(49) \text{ MeV}. \quad (10)$$

From the previous result, one can deduce the value of the mass-splitting between the 1st radial excitation and the lowest mass ground state: $M'_{Y_{bd}} - M_{Y_{bd}} \approx M'_{Y_{cd}} - M_{Y_{cd}} \approx 250 \text{ MeV}$, which are (almost) heavy-flavour independent and also smaller than the one of the bottomium splitting ($\sim 560 \text{ MeV}$).

In the following, we shall let the current mixing parameter b , as defined in Eq. (5), free and study its effect on the results obtained in Eqs. (9) and (10). In so doing, we fix the values of τ around the τ -stability point and t_c around the intersection point of the LSR and FESR. The results of the analysis are shown in Fig. 2. We notice that the results are optimal at the value $b = 0$.

For completing the analysis of the effect of b , we also study the decay constant $f_{Y_{Qd}}$ defined as: $\langle 0 | j_{4q}^\mu | Y_{Qd} \rangle = f_{Y_{Qd}} M_{Y_{Qd}}^\mu \epsilon^\mu$. Doing the analysis, giving $M_{Y_{Qd}}$ and the corresponding t_c obtained above, one can deduce the optimal values at $b = 0$:

$$f_{Y_{cd}} \simeq 0.08 \text{ MeV} \quad \text{and} \quad f_{Y_{bd}} \simeq 0.03 \text{ MeV}, \quad (11)$$

which are much smaller than $f_\pi = 132 \text{ MeV}$, $f_\rho \simeq 215 \text{ MeV}$ and $f_D \simeq f_B = 203 \text{ MeV}$ [38]. One can also note that the decay constant decreases like $1/M_Q$ which can be tested in HQET or/and lattice QCD.

• $SU(3)$ breaking for $M_{Y_{Qs}}$ from DRSR

We study the ratio $M_{Y_{Qs}}/M_{Y_{Qd}}$ using DRSR:

$$r_{sd}^Q \equiv \sqrt{\mathcal{R}_{Qs}^{LSR}} / \sqrt{\mathcal{R}_{Qd}^{LSR}} \quad \text{where} \quad Q \equiv c, b. \quad (12)$$

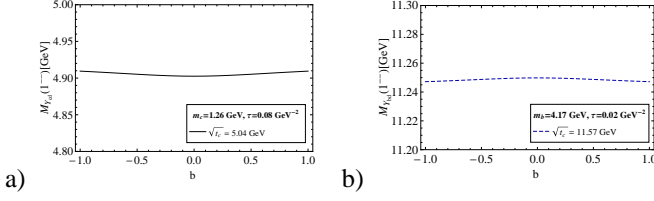


Figure 2: **a)** b -behaviour of $M_{Y_{cd}}$ for given values of τ and t_c and for $m_c = 1.26$ GeV; **b)** the same as a) but for $M_{Y_{bd}}$ and for $m_b = 4.17$ GeV.

Extracting the values at τ - and t_c -stabilities points we can deduce, respectively for $\sqrt{t_c} = 5.1$ and 11.6 GeV:

$$\begin{aligned} r_{sd}^c &= 1.018(1)_{m_c(5)m_s(2)_\kappa(2)\bar{u}u(1)_\rho}, \\ r_{sd}^b &= 1.007(0.5)_{m_b(2)m_s(0.5)_\kappa(1)\bar{u}u(0.3)_\rho}. \end{aligned} \quad (13)$$

Using the results for Y_{Qd} in Eqs. (9) and (10) and the values of the $SU(3)$ breaking ratio in Eq. (13), we can deduce the mass of the Y_{Qs} state in MeV:

$$M_{Y_{cs}} = 4900(67), \quad M_{Y_{bs}} = 11334(55), \quad (14)$$

leading to the $SU(3)$ mass-splitting: $\Delta M_{sd}^{Y_c} \approx 87$ MeV $\approx \Delta M_{sd}^{Y_b} \approx 78$ MeV, which is also heavy-flavour independent.

4. 1^{--} molecule masses from QSSR

• The $\bar{D}_{d(s)}^* D_{d(s)}$ and $\bar{B}_{d(s)}^* B_{d(s)}$ molecules ¹

Like in the previous case, we use LSR and FESR for studying the masses of the $\bar{D}_d^* D_d$ and $\bar{B}_d^* B_d$ and DRSR for studying the $SU(3)$ breaking ratios:

$$r_{sd}^D \equiv M_{D_d^* D_s} / M_{D_d^* D_d}, \quad r_{sd}^B \equiv M_{B_d^* B_s} / M_{B_d^* B_d}. \quad (15)$$

Using the sets ($m_c = 1.26$ GeV, $\sqrt{t_c} = 5.58$ GeV) and ($m_b = 4.17$ GeV, $\sqrt{t_c} = 11.64(3)$ GeV) common solutions of LSR and FESR, one can deduce in MeV:

$$\begin{aligned} M_{D_d^* D_d} &= 5268(24), \quad M_{B_d^* B_d} = 11302(30), \\ r_{sd}^D &= 1.018(1)_{m_c(4)m_s(0.8)_\kappa(0.5)\bar{u}u(0.2)_\rho(0.1)_{G^3}}, \\ r_{sd}^B &= 1.006(1)_{m_b(2)m_s(1)_\kappa(0.5)\bar{u}u(0.2)_\rho(0.1)_{G^3}}. \end{aligned} \quad (16)$$

Using the previous results in Eq. (16), one obtains in MeV:

$$M_{D_s^* D_s} = 5363(33), \quad M_{B_s^* B_s} = 11370(40), \quad (17)$$

corresponding to a $SU(3)$ mass-splitting: $\Delta M_{sd}^{DD^*} \approx 95$ MeV $\approx \Delta M_{sd}^{BB^*} \approx 68$ MeV.

• The $J/\psi S_2$ and ΥS_2 molecules

Combining LSR and FESR, we consider the mass of the $J/\psi S_2$ and ΥS_2 molecules in a colour singlet combination, where $S_2 \equiv \bar{u}u + \bar{d}d$ is a scalar meson. We work with the LO QCD expression obtained in [16]. Then we analyze the t_c -behaviour of different τ -extremas, from which we can deduce, for the running quark masses for $\sqrt{t_c} = 5.30(2)$ and $10.23(3)$ GeV, in MeV:

$$M_{J/\psi S_2} = 5002(31), \quad M_{\Upsilon S_2} = 10015(33). \quad (18)$$

The splitting (in units of MeV) with the first radial excitation approximately given by $\sqrt{t_c}$ is:

$$M'_{J/\psi S_2} - M_{J/\psi S_2} \approx 298, \quad M'_{\Upsilon S_2} - M_{\Upsilon S_2} \approx 213. \quad (19)$$

In the same way, we analyse the τ and t_c behaviours of the $SU(3)$ breaking ratios, from which, we can deduce:

$$\begin{aligned} r_{sd}^{J/\psi} &\equiv M_{J/\psi S_3} / M_{J/\psi S_2} = 1.022(0.2)_{m_c(5)m_s(2)_\kappa}, \\ r_{sd}^{\Upsilon} &\equiv M_{\Upsilon S_3} / M_{\Upsilon S_2} = 1.011(1)_{m_b(2)m_s(0.2)_\kappa}, \end{aligned} \quad (20)$$

where $S_3 \equiv \bar{s}s$ is a scalar meson. Then, we obtain in MeV:

$$M_{J/\psi S_3} = 5112(41), \quad M_{\Upsilon S_3} = 10125(40), \quad (21)$$

corresponding to the $SU(3)$ mass-splittings: $\Delta M_{sd}^{J/\psi} \approx \Delta M_{sd}^{\Upsilon} \approx 110$ MeV. Doing the same exercise for the octet current, we deduce the results in Table 2 where the molecule associated to the octet current is 100 (resp. 250) MeV above the one of the singlet current for J/ψ (resp. Υ) contrary to the 1^{++} case discussed in [3]. The ratio of $SU(3)$ breakings are respectively 1.022(5) and 1.010(2) in the c and b channels which are comparable with the ones in Eq.(20).

5. 0^{++} four-quark and molecule masses from QSSR

• Y_{Qd}^0 mass and decay constant from LSR and FESR

We do the analysis of the Y_{cd}^0 and Y_{bd}^0 masses using LSR and FESR match. We work with the current mixing parameter $b = 0$ from which we deduce in MeV, for the running quark masses, and respectively for $\sqrt{t_c} = 6.5$ and 13.0 GeV:

$$M_{Y_{cd}^0} = 6125(51) \text{ MeV}, \quad M_{Y_{bd}^0} = 12542(43) \text{ MeV}. \quad (22)$$

One can notice that the splittings between the lowest ground state and the 1st radial excitation approximately given by $\sqrt{t_c}$ is: $M'_{Y_{cd}^0} - M_{Y_{cd}^0} \approx 375$ MeV, $M'_{Y_{bd}^0} - M_{Y_{bd}^0} \approx 464$ MeV, which is larger than the ones of the 1^{--} states, comparable with the ones of the J/ψ and Υ , and are (almost) heavy-flavour independent. For completeness, we calculate the sum rule for the decay constants from which we deduce:

$$f_{Y_{cd}^0} \approx 0.12 \text{ MeV} \quad \text{and} \quad f_{Y_{bd}^0} \approx 0.03 \text{ MeV}, \quad (23)$$

which are comparable with the ones of the spin 1 case.

• $SU(3)$ breaking for $M_{Y_{Qs}}^0$ from DRSR

Analyzing the τ and t_c behaviours of the $SU(3)$, we deduce:

$$\begin{aligned} r_{sd}^{0c} &= 1.011(2)_{m_c(3.8)m_s(1.4)_\kappa(1)\bar{u}u(0.7)_\rho}, \\ r_{sd}^{0b} &= 1.004(1)_{m_c(1.7)m_s(0.3)_\kappa}, \end{aligned} \quad (24)$$

leading to the masses in MeV and theirs respective $SU(3)$ mass-splittings:

$$\begin{aligned} M_{Y_{cs}^0} &= 6192(59), \quad M_{Y_{bs}^0} = 12592(50), \\ \Delta M_{sd}^{Y_c^0} &\approx 67 \approx \Delta M_{sd}^{Y_b^0} \approx 50 \text{ MeV}. \end{aligned} \quad (25)$$

¹Hereafter, for simplicity D and B denote the scalar D_0^* and B_0^* mesons.

• $M_{D_d D_d}$ and $M_{B_d B_d}$ from LSR and FESR

We consider as a final result the one corresponding to the running masses, for $\sqrt{t_c} = 6.25(3)$ and 12.02 GeV:

$$M_{D_d D_d} = 5955(48) \text{ MeV}, \quad M_{B_d B_d} = 11750(40) \text{ MeV}. \quad (26)$$

The splittings (in MeV) between the lowest ground state and the 1st radial excitation, approximately given by $\sqrt{t_c}$, is:

$$M'_{D_d D_d} - M_{D_d D_d} \approx 290, \quad M'_{B_d B_d} - M_{B_d B_d} \approx 270, \quad (27)$$

which, like in the case of the 1^{--} states are smaller than the ones of the J/ψ and Υ , and almost heavy-flavour independent.

• $SU(3)$ breaking for $M_{\bar{D}_s D_s}$ and $M_{\bar{B}_s B_s}$ from DRSR

Calculating the $SU(3)$ mass ratios for the molecules $\bar{D}_s D_s$ and $\bar{B}_s B_s$, considering different values of t_c and the t_c behaviour of their τ -extremas, we deduce:

$$\begin{aligned} r_{sd}^{OD} &\equiv M_{D_s D_s} / M_{D_d D_d} = 1.015(1)_{m_c}(4)m_s(2)_\kappa(1)\bar{u}u(0.5)_\rho, \\ r_{sd}^{OB} &\equiv M_{B_s B_s} / M_{B_d B_d} = 1.008(1)_{m_c}(4)m_s(2)_\kappa(1)\bar{u}u(0.5)_\rho. \end{aligned}$$

Using the previous values of $M_{D_d D_d}$ and $M_{B_d B_d}$, we deduce:

$$M_{D_s D_s} = 6044(56) \text{ MeV}, \quad M_{B_s B_s} = 11844(50) \text{ MeV}, \quad (28)$$

which corresponds to a $SU(3)$ splitting:

$$\Delta M_{sd}^{DD} \approx 89 \text{ MeV} \approx \Delta M_{sd}^{BB} \approx 94 \text{ MeV}. \quad (29)$$

Table 2: Masses of the four-quark and molecule states from the present analysis combining Laplace (LSR) and Finite Energy (FESR).

States		States	
<i>Four-quarks</i>	1^{--}		0^{++}
Y_{cd}	4818(27)	Y_{cd}^0	6125(51)
Y_{cs}	4900(67)	Y_{cs}^0	6192(59)
Y_{bd}	11256(49)	Y_{bd}^0	12542(43)
Y_{bs}	11334(55)	Y_{bs}^0	12592(50)
<i>Molecules</i>	1^{--}		0^{++}
$\overline{D}_q D_d$	5268(24)	$\overline{D}_d D_d$	5955(48)
$\overline{D}_s D_s$	5363(33)	$\overline{D}_s D_s$	6044(56)
$\overline{B}_d B_d$	11302(30)	$\overline{B}_d B_d$	11750(40)
$\overline{B}_s B_s$	11370(40)	$\overline{B}_s B_s$	11844(50)
<i>Singlet current</i>	1^{--}	<i>Octet current</i>	1^{--}
$J/\psi S_2$	5002(31)		5118(29)
$J/\psi S_3$	5112(41)		5231(40)
ΥS_2	10015(33)		10268(28)
ΥS_3	10125(40)		10371(45)

6. Summary and conclusions

• The three $Y_c(4260, 4360, 4660)$ 1^{--} experimental candidates are too low for being pure four-quark or/and molecule $\bar{D}D^*$ and $J/\psi S_2$ states but can result from their mixings. The $Y_b(10890)$ is lower than the predicted values of the four-quark and $\bar{B}B^*$ molecule masses but heavier than the predicted ΥS_2 and ΥS_3 molecule states. Our results may indicate that some other exotic structure of these states are not excluded.

• For the 1^{--} , there is a regularity of about (250-300) MeV for the value of the mass-splittings between the lowest ground state and the 1st radial excitation roughly approximated by the value

of the continuum threshold $\sqrt{t_c}$ at which the LSR and FESR match. These mass-splittings are (almost) flavour-independent and are much smaller than the ones of 500 MeV of ordinary charmonium and bottomium states.

• There is also a regularity of about 50–90 MeV for the $SU(3)$ mass-splittings of the different states which are also (almost) flavour-independent.

• The spin 0 states are much more heavier (≥ 400 MeV) than the spin 1 states, like in the case of hybrid states [5].

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